Aspects of Precise Orbit Determination for Geodetic and Altimetric Missions: 
Current Issues and Future Trends

Philip Moore
Department of Geomatics
University of Newcastle upon Tyne
Newcastle upon Tyne
NE1 7RU
Philip.Moore@Newcastle.ac.uk
Improvements in precise orbit determination for altimeter satellites associated with

1. Improved accuracy and global distribution of tracking data e.g. DORIS, GPS
2. Gravity field enhancement
3. Surface force modelling
4. Orbit determination techniques introducing empirical/stochastic parameters for surface force and other modelling errors
5. Use of altimetry in the form of single (SXO) or dual satellite crossovers (DXO) as additional tracking data in both dynamic/reduced dynamic procedures and in empirical enhancement of less accurate orbit via cubic splines.
Orbit determination techniques introducing empirical/stochastic parameters for surface force and other modelling errors

Non-gravitational forces
- aerodynamic effects
- direct solar radiation pressure
- earth reflected and infrared radiation
- thermal forces
- charged particle drag

Surface force modelling
- satellite surface macro models e.g. box wing for TOPEX/Poseidon
- modelling of geometry and surface properties (area, outward pointing normal)
- modelling of momentum exchange between spacecraft surface and atmospheric molecules and incoming radiation
- environmental models for neutral air density models (MSIS83, DTM98), earth and albedo distribution
- eclipse model for direct solar radiation pressure (penumbra, umbra)
- surface composition (reflection characteristics - absorption, transmission, reflection), thermal properties (internal and external heating)
Surface Forces: Sample of CHAMP accelerometer data Day 144 2001 (24 May)
DFT analysis of accelerometer data: radial
DFT analysis of accelerometer data: along-track
DFT analysis of accelerometer data: cross-track
Difference between CHAMP accelerometer and modelled along-track accelerations
Reduced Dynamic procedure: multiple empirical parameters introduced to reduce tracking data residuals without concern about source of modelling error

With high density of tracking data and global coverage of orbit common practice is to solve for empirical corrections to poor dynamic modelling, e.g. multiple drag scale factors, 1cy/rev along/cross track accelerations. Parameters are assumed uncorrelated and solved in normal batch solutions.

If geometric data is near sufficient to support orbit determination then importance of dynamic models can be reduced and a large number of additional parameters can be introduced and recovered

Two approaches:
1. Utilise stochastic parameters
2. Introduce multiple parameters with correlations
Multiple parameters with correlations

Orbital errors typically 1cy/rev with amplitude slowly varying with time

In dynamic procedure 1 cy/rev empirical accelerations along-track and cross-track over say a 24hr time span are introduced to absorb mis-modelling in force model, i.e. 4 parameters per day.

In reduced dynamic models the procedure is generalised to empirical accelerations every say ¼ revolution for example, i.e. 25min for TOPEX/Poseidon.

T/P 127 revs per 10 day cycle \( \Rightarrow \) 508*4 parameters
Let along-track acceleration be \( A_i \cos \omega t + B_i \sin \omega t \)

then add pseudo-observations

\[
A_i = A_j \quad \text{with weight } w_{i,j}
\]

Where

\[
W_{i,j} = \frac{e^{-\left| \frac{t_i - t_j}{\tau} \right|}}{\sigma^2 e^{\frac{t_i - t_j}{\tau}}}
\]

- \( t_i \) reference time tag for interval i; \( t_{i+1} - t_i = \Delta t \)
- \( \tau \) correlation time
- \( \sigma \) standard deviation for the amplitudes \( A_i \)
Reduced dynamic orbits: example 1: Jason-1

• **Dynamic orbit**
  – State vector:
    • Initial position/velocity
    • 6hr drag scale factors
    • Daily empirical accelerations along/cross track
    • Bias and trop. Scale factor for each DORIS pass
  – Tracking
    • SLR, DORIS (SXO – but not used in orbit determination)

• **Reduced dynamic orbit**
  – $\Delta t = 1$ hr
    • 240 sets of (4) empirical parameters for 10 day arc
  – $\tau = 5$ hr (correlation time)
  – $\sigma = 5.0d-5$ Mm/day**2
Reduced dynamic orbits:  
example 1: Jason-1

<table>
<thead>
<tr>
<th>Cycle strategy</th>
<th>Obs. RMS</th>
<th>Eph RMS/mean</th>
<th>SXO RMS/mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SLR (cm)</td>
<td>Radial (diff with GSFC) cm</td>
<td>Radial (diff with GSFC) cm</td>
</tr>
<tr>
<td></td>
<td>DORIS (mm/s)</td>
<td>cross along</td>
<td>mean</td>
</tr>
<tr>
<td>4 dyn</td>
<td>1.95</td>
<td>1.84/-0.1</td>
<td>1.84/-0.1</td>
</tr>
<tr>
<td>r.dyn</td>
<td>1.06</td>
<td>1.65/-0.1</td>
<td>1.65/-0.1</td>
</tr>
<tr>
<td>8 dyn</td>
<td>2.26</td>
<td>1.55/-0.1</td>
<td>1.55/-0.1</td>
</tr>
<tr>
<td>r.dyn</td>
<td>1.20</td>
<td>1.56/-0.1</td>
<td>1.56/-0.1</td>
</tr>
<tr>
<td>9 dyn</td>
<td>2.43</td>
<td>1.47/-0.1</td>
<td>1.47/-0.1</td>
</tr>
<tr>
<td>r.dyn</td>
<td>1.66</td>
<td>1.54/-0.1</td>
<td>1.54/-0.1</td>
</tr>
</tbody>
</table>

Note: SXO data not included in orbit determination  
DORIS freq offset/trop bias held fixed in reduced dyn. orbit
Reduced dynamic orbits: example 2: GFO

• Dynamic orbit (19 May 2000 – 24 May 2000)
  – State vector:
    • Initial position/velocity
    • 8hr drag scale factors
    • 1 set empirical accelerations along/cross track
    • Range and time bias for GFO altimetry
  – Tracking
    • SLR, SXO, DXO with TOPEX/Poseidon
    • TOPEX/Poseidon orbit held fixed

• Reduced dynamic orbit
  – \( \Delta t = 0.5 \) hr
    • 120 sets of (4) empirical parameters
  – \( \tau = 1 \) hr (correlation time)
  – \( \sigma = 1.0 \times 10^{-4} \) Mm/day**2
### Reduced dynamic orbits: example 2: GFO

<table>
<thead>
<tr>
<th>DATA</th>
<th>Obs. RMS</th>
<th>Eph RMS/mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(cm)</td>
<td>Radial cross along (diff with GSFC) cm</td>
</tr>
<tr>
<td>Dyn SLR</td>
<td>6.13</td>
<td>6.97/0.08 23.49/-1.2 36.87/-3.1</td>
</tr>
<tr>
<td>Dyn SLR SXO</td>
<td>6.45</td>
<td>3.83/0.03 12.71/-1.2 25.33/-5.1</td>
</tr>
<tr>
<td>Dyn SLR SXO DXO</td>
<td>7.03</td>
<td>3.35/0.01 14.43/-1.5 24.10/-4.6</td>
</tr>
<tr>
<td>R. Dyn SLR SXO DXO</td>
<td>5.19</td>
<td>3.36/0.03 13.65/-1.5 23.71/-0.4</td>
</tr>
</tbody>
</table>
Precise Orbit Determination - Current status

- Reduced dynamic Topex/Poseidon orbits: SXO residuals ~ 6-7cm
- GFO and ERS orbits:
  - SXO residuals ~ 7.5cm
  - DXO residuals ~ 7.5cm
Developments in SLR

- Observing stations submit data sub daily for MOE
- SLR2000: fully automated
- Short-arc corrections during periods of quasi-simultaneous tracking by 3 or more SLR stations applied to MOE gives radial accuracy of 10mm or better

2002-Jun-19

Station Grasse
Satellite Envisat

7835 Grasse
7824 San Fernando
7839 Graz
7848 Ajaccio
Developments in GPS

- JPL developing space borne real time orbit determination capability: Real Time Gipsy (RTG)
- RTG used on ground to compute GPS s/c orbits, 1s clock corrections and tropospheric delay parameters
- Differential corrections relayed via internet (broadband) wireless telephones such as iridium system and Inmarsat (between lat 75N and S)

<table>
<thead>
<tr>
<th>Satellite</th>
<th>Altitude (km)</th>
<th>Radial rms (cm)</th>
<th>Cross-track rms (cm)</th>
<th>Along-track rms (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jason-1</td>
<td>1340</td>
<td>7</td>
<td>5</td>
<td>17</td>
</tr>
<tr>
<td>CHAMP*</td>
<td>450</td>
<td>11</td>
<td>8</td>
<td>26</td>
</tr>
</tbody>
</table>

* Deduced from CHAMP 30cm 3D rms and Jason-1 coordinate accuracies
Simulation of orbital enhancement using low accuracy Cartesian positioning from GPS and DXO/SXO data

• **Five day GFO arc**
  – Cartesian positioning from reduced dynamic orbit
  – random noise, \( n \), added to \( x, y, z \): \( n \in N(\mu, \sigma) \)
  – SXO differences from GFO altimetry
  – DXO differences with T/P
  – 5 day maximum time difference for SXO/DXO data

• **Dynamic solution**
  – solution vector: \( \mathbf{X}, \dot{\mathbf{X}}, \ddot{\mathbf{X}} \); 8hr drag factors;
    1cy/rev per day accel along/cross rel. bias (DXO)
GFO SXO locations: 19-24 May 2000;
(5 day maximum difference between epochs)
GFO-T/P DXO locations: 19-24 May 2000
(5 day maximum difference between epochs)
• Simulations

<table>
<thead>
<tr>
<th>$\mu$, $\sigma$</th>
<th>cart</th>
<th>cart+SXO+DXO</th>
<th>wt/xyz</th>
<th>wt/zo</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu=0$, $\sigma=1$</td>
<td>$3.48/3.61/2.79$</td>
<td>$2.96/3.53/2.56$</td>
<td>100</td>
<td>10</td>
</tr>
<tr>
<td>$\mu=0$, $\sigma=2$</td>
<td>$7.43/7.20/5.90$</td>
<td>$5.41/6.48/4.64$</td>
<td>200</td>
<td>10</td>
</tr>
<tr>
<td>$\mu=0$, $\sigma=3$</td>
<td>$11.61/10.96/9.18$</td>
<td>$7.76/9.44/6.60$</td>
<td>300</td>
<td>10</td>
</tr>
<tr>
<td>$\mu=0.1$, $\sigma=1$</td>
<td>$6.12/5.60/4.36$</td>
<td>$2.80/3.50/2.49$</td>
<td>100</td>
<td>10</td>
</tr>
<tr>
<td>$\mu=0.5$, $\sigma=1$</td>
<td>$30.32/25.20/19.93$</td>
<td>$4.43/4.67/3.38$</td>
<td>150</td>
<td>10</td>
</tr>
<tr>
<td>$\mu=0.5$, $\sigma=5$</td>
<td>$32.39/29.06/23.59$</td>
<td>$11.51/15.32/10.19$</td>
<td>500</td>
<td>10</td>
</tr>
</tbody>
</table>

$\mu$, $\sigma$ in m;
radial/cross/along differences against $N(0,0)$ in cm
weights in cm
Summary:

– SXO/DXO data can recover radial positioning to $3(5)(8)\text{cm}$ for random errors with $\sigma = 1(2)(3)\text{m}$

– systematic errors (ie $\mu \neq 0$) in $x$, $y$ not significant; $z$ highly significant

• results for $N(\mu, \sigma)$ with $\mu = 0.5\text{m}$, $\sigma = 5\text{m}$

<table>
<thead>
<tr>
<th>coord</th>
<th>cart. tracking</th>
<th></th>
<th>cart+SXO+DXO</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>rms (cm)</td>
<td>mean (cm)</td>
<td>rms (cm)</td>
<td>mean (cm)</td>
</tr>
<tr>
<td>x</td>
<td>503.1</td>
<td>52.2</td>
<td>502.9</td>
<td>52.6</td>
</tr>
<tr>
<td>y</td>
<td>500.0</td>
<td>49.9</td>
<td>499.6</td>
<td>48.8</td>
</tr>
<tr>
<td>z</td>
<td>501.9</td>
<td>9.6</td>
<td>504.0</td>
<td>45.2</td>
</tr>
</tbody>
</table>
Current Issues

Current issues: IGGOS (integrated Global Geodetic Observing System)

- static gravity field modelling - dedicated gravity field missions (CHAMP, GRACE, GOCE)
- time varying field and geocentre - seasonal variations etc due to mass redistributions in atmosphere, hydrosphere, cryosphere and oceans. Geophysical models inadequate for task especially in hydrology. GRACE will provide temporal variations which can be applied retrospectively at annual cycle
- neutral air-density - static models inadequate for dynamic processes in atmosphere. Accelerometers on CHAMP and GRACE will give further data at heights of 450 km but extrapolation to higher altitudes problematic. Need empirical parameters to absorb deficiencies.
- Requirement for multi tracking data types. Lesson with GFO. Calibration and offsets data type specific. SLR proved crucial to quantify accuracy of CHAMP GPS orbits.
- Integration of multi-satellite altimetry requires either absolute calibration (IGGOS; GPS, tide gauge or buoys, SLR, radiometers etc) or cross calibration using DXO analyses and/or relative calibration using tide gauge data.
- Real time GPS v Diode